JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 22, No. 3, May–June 1999

Engineering Notes

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Controller Design of Periodic Time-Varying Systems via Time-Invariant Methods

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I. Introduction

RECENTLY, Sinha and Joseph[1] considered the pole assignment problem in a linear periodic time-varying system using time-varying state feedback. The authors suggested designing the output feedback controller and estimator with the Floquet transformed time-invariant system. However, the method developed to design the time-varying feedback gains has some problems and cannot always guarantee the stability of the original linear time-varying system. Here, we will revise the algorithm and develop a new control technique of the linear time-varying system based on the output feedback control algorithm with time-varying control gains.

II. Controller Design Formulation

A. Full State Feedback Controller Design

First, we introduce the technique to design the time-varying controller with time-varying feedback gains that are based on the Lyapunov–Floquet transformed system. Let us consider the linear periodic time-varying system (LPTS) described by the following state equation:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{1}$$

$$y(t) = C(t)x(t) \tag{2}$$

Applying the Lyapunov–Floquet transformation, one can write Eqs. (1) and (2) as

$$\dot{z}(t) = \bar{A}z(t) + \bar{B}(t)u(t) \tag{3}$$

$$y(t) = \bar{C}(t)z(t) \tag{4}$$

where $\bar{A} = P^{-1}(t)A(t)P(t) - P^{-1}(t)\dot{P}(t)$ is a constant matrix, and $\bar{B}(t) = P^{-1}(t)B(t)$ and $\bar{C}(t) = C(t)P(t)$ are periodic time-varying matrices.

Now, consider the ideal system of the form

$$\dot{z}^*(t) = \bar{A}z^*(t) + \bar{B}u^*(t) \tag{5}$$

$$y^*(t) = \bar{C}z^*(t) \tag{6}$$

where $\bar{B}$ and $\bar{C}$ are constant matrices such that $(\bar{A}, \bar{B})$ is a controllable and $(\bar{A}, \bar{C})$ is an observable pair, respectively.

Received July 15, 1997; revision received Aug. 21, 1998; accepted for publication Sept. 2, 1998.

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Then the full state feedback control law of the linear time-invariant system (5) is

$$u^*(t) = \bar{G}z^*(t) \tag{7}$$

where $\bar{G}$ is a constant gain matrix that can be designed to make the ideal system (5) stable.

To get the full state feedback control law of the original LPTS, let us define the error signal

$$e^*(t) \equiv z(t) - z^*(t) \tag{8}$$

Then the error system from Eqs. (3) and (5) can be represented as

$$\begin{aligned}\dot{e}^*(t) &= [\bar{A}z(t) - \bar{B}(t)u(t)] - [\bar{A}z^*(t) + \bar{B}u^*(t)] \\ &= \bar{A}e^*(t) + [\bar{B}(t)u(t) - \bar{B}u^*(t)] \\ &= (\bar{A} + \bar{B}\bar{G})e^*(t) + [\bar{B}(t)u(t) - \bar{B}\bar{G}z(t)]\end{aligned} \tag{9}$$

To make the error system (8) stable, the second term of the equation must be zero, i.e., $\bar{B}(t)u(t) - \bar{B}\bar{G}z(t) = 0$. If we assume that $\bar{B}(t)$ has a full rank for a time period T, then we can get the full state feedback control law of LPTS:

$$u(t) = [\bar{B}^{\#}(t)\bar{B}\bar{G}]z(t) = \bar{G}(t)z(t) \tag{10}$$

where $\bar{B}^{\#}(t) \equiv [\bar{B}^T(t)\bar{B}(t)]^{-1}\bar{B}^T(t)$ is the Moore–Penrose pseudoinverse that satisfies $\bar{B}^{\#}(t)\bar{B}(t) = I$ and $\bar{G}(t)$ is a time-varying feedback gain matrix.

Now, to check the closed-loop stability of the ideal system, substitute Eq. (10) into Eq. (9):

$$\begin{aligned}\dot{e}^*(t) &= (\bar{A} + \bar{B}\bar{G})e^*(t) + [\bar{B}(t)\bar{B}^{\#}(t)\bar{B}\bar{G} - \bar{B}\bar{G}]z(t) \\ &= (\bar{A} + \bar{B}\bar{G})e^*(t) + [\bar{B}(t)\bar{B}^{\#}(t) - I]\bar{B}\bar{G}z(t) \\ &= (\bar{A} + \bar{B}\bar{G})e^*(t) + \Delta_B(t)\bar{B}\bar{G}z(t) \\ &= [\bar{A} + \bar{B}\bar{G} - \Delta_B(t)\bar{B}\bar{G}]e^*(t) + \Delta_B(t)\bar{B}\bar{G}z^*(t)\end{aligned}$$

where $\Delta_B(t)$ is given by $\bar{B}(t)\bar{B}^{\#}(t) = I - \Delta_B(t)$.

So the whole closed-loop system can be written as

$$\begin{aligned}\begin{bmatrix}\dot{e}^* \\ \dot{z}^*\end{bmatrix} &= \left(\begin{bmatrix}\bar{A} + \bar{B}\bar{G} & 0 \\ 0 & \bar{A} + \bar{B}\bar{G}\end{bmatrix} - \begin{bmatrix}\Delta_B(t)\bar{B}\bar{G} & \Delta_B(t)\bar{B}\bar{G} \\ 0 & 0\end{bmatrix}\right) \cdot \begin{bmatrix}e^* \\ z^*\end{bmatrix} \\ &= [A_o + \Delta A(t)] \cdot \begin{bmatrix}e^* \\ z^*\end{bmatrix}\end{aligned} \tag{11}$$

Theorem[2]: Consider an LPTS

$$\dot{w}(t) = A_C(t)w(t) = [A_o + \Delta A(t)]w(t) \tag{12}$$

where A_o is a constant, stable matrix and $\Delta A(t)$ is a periodic time-varying matrix that is bounded by ε, i.e., $\|\Delta A(t)\| < \varepsilon$.

Then the fundamental matrix of Eq. (12) is bounded by

$$\|\Phi_C(t, 0)\| \le K_o \cdot e^{-\sigma_c t}$$

where $\sigma_c \equiv \sigma_o - K_o\varepsilon$ and $\|e^{A_o t}\| \le K_o \cdot e^{-\sigma_o t}$.

Also if ε is bounded by σ_o/K_o, i.e., $0 < \varepsilon < \sigma_o/K_o$, then the system is exponentially stable.

Corollary[2]*:* In Eq. (11), if $\|\Delta_B(t)\|$ is sufficiently small, then $\|z(t)\|$ is bounded by $\|z(t)\| \le K_z e^{-\sigma_c t}$, where $\sigma_c \equiv \sigma_o - \varepsilon K_o$ and $K_z = 4K_o \cdot \|e_*(0)\|$.

III. Controller Design with State Estimator

Up to now, we have discussed the design of the time-varying feedback gain matrix $\bar{G}(t)$. Based on this new approach, we can design the time-varying observer much more easily than with the existing method.

We begin by considering the Lyapunov–Floquet transformed system with the state estimator:

$$\dot{\hat{z}}(t) = \bar{A}\hat{z}(t) + \bar{B}(t)u(t) + \bar{K}(t)[y(t) - \hat{y}(t)] \tag{13}$$

$$\hat{y}(t) = \bar{C}(t)\hat{z}(t) \tag{14}$$

Also, redefine the ideal system (5) and (6) in the form

$$\dot{\hat{z}}^*(t) = \bar{A}\hat{z}^*(t) + \bar{B}u^*(t) + \bar{K}[y^*(t) - \hat{y}^*(t)] \tag{15}$$

$$\hat{y}^*(t) = \bar{C}\hat{z}^*(t) \tag{16}$$

where gain matrices $\bar{G}$ and $\bar{K}$ can be determined by using pole placement or the linear quadratic regulator algorithm and the control input $u^*(t)$ is defined as $u^*(t) \equiv \bar{G}\hat{z}^*(t)$.

Now, we focus our attention on getting the time-varying observer gain matrix $\bar{K}(t)$. Let us define the error signals

$$e^*(t) = z(t) - z^*(t) \tag{17}$$

$$\hat{e}^*(t) = \hat{z}(t) - \hat{z}^*(t) \tag{18}$$

Differentiating Eq. (18), we can get

$$\begin{aligned}\dot{\hat{e}}^*(t) &= \dot{\hat{z}}(t) - \dot{\hat{z}}^*(t)\\ &= \bar{A}\hat{z}(t) + \bar{B}(t)u(t) + \bar{K}(t)[y(t) - \hat{y}(t)]\\ &\quad - \bar{A}\hat{z}^*(t) - \bar{B}u^*(t) - \bar{K}[y^*(t) - \hat{y}^*(t)]\end{aligned} \tag{19}$$

By substituting Eqs. (14) and (16), we can rewrite Eq. (19) in the form

$$\begin{aligned}\dot{\hat{e}}^*(t) &= (\bar{A} - \bar{K}\bar{C})\hat{e}^*(t) + [\bar{B}(t)u(t) - \bar{B}\bar{G}\hat{z}^*(t)] - \bar{K}\bar{C}e^*(t)\\ &\quad + [\bar{K}(t)\bar{C}(t) - \bar{K}\bar{C}] \cdot [z(t) - \hat{z}(t)]\end{aligned} \tag{20}$$

To make the system equation (20) stable, the last term of the equation must be zero, which implies that

$$\bar{K}(t)\bar{C}(t) - \bar{K}\bar{C} = 0 \Rightarrow \bar{K}(t) = \bar{K}\bar{C}\bar{C}^{\#}(t) \tag{21}$$

where $\bar{C}^{\#}(t)$ is the Moore–Penrose pseudoinverse that satisfies $\bar{C}^{\#}(t)\bar{C}(t) = I$.

We now examine the stability of the whole closed-loop system with time-varying gain matrices $\bar{G}(t)$ and $\bar{K}(t)$. The resulting closed-loop system of Eqs. (3), (4), (18), and (19) is written as

$$\begin{aligned}\begin{bmatrix}\dot{z}\\ \dot{\hat{z}}\end{bmatrix} &= \begin{bmatrix}\bar{A} & \bar{B}(t)\bar{G}(t)\\ \bar{K}(t)\bar{C}(t) & A + \bar{B}(t)\bar{G}(t) - \bar{K}(t)\bar{C}(t)\end{bmatrix} \cdot \begin{bmatrix} z\\ \hat{z}\end{bmatrix}\\ &= \begin{bmatrix}\bar{A} & \bar{B}(t)\bar{B}^{\#}(t)\bar{B}\bar{G}\\ \bar{K}\bar{C}\bar{C}^{\#}(t)\bar{C}(t) & \bar{A} + \bar{B}(t)\bar{B}^{\#}(t)\bar{B}\bar{G} - \bar{K}\bar{C}\bar{C}^{\#}(t)\bar{C}(t)\end{bmatrix} \cdot \begin{bmatrix} z\\ \hat{z}\end{bmatrix}\\ &= \left(\begin{bmatrix}\bar{A} & \bar{B}\bar{G}\\ \bar{K}\bar{C} & \bar{A} + \bar{B}\bar{G} - \bar{K}\bar{C}\end{bmatrix}\right.\\ &\quad + \left.\begin{bmatrix}0 & -\Delta_B(t)BG\\ -KC\Delta_C(t) & -\Delta_B(t)BG + KC\Delta_C(t)\end{bmatrix}\right) \cdot \begin{bmatrix} z\\ \hat{z}\end{bmatrix}\\ &= [\bar{A}_o + \Delta\bar{A}(t)] \cdot \begin{bmatrix} z\\ \hat{z}\end{bmatrix}\end{aligned} \tag{22}$$

where $\bar{A}_o$ is stable and $\Delta_B(t)$ and $\Delta_C(t)$ are given by $\bar{B}(t)\bar{B}^{\#}(t) = I - \Delta_B(t)$ and $\bar{C}^{\#}(t)\bar{C}(t) = I - \Delta_C(t)$, respectively.

Therefore, by the theorem, if $\Delta A(t)$ is small, the whole closed-loop system is stable.

IV. Numerical Results

Consider the commutative LPTS by the set of equations as shown next:

$$\dot{x}(t) = \omega \cdot \begin{bmatrix}-1 + a \cdot \cos^2(\omega t) & 1 - a \cdot \cos(\omega t) \cdot \sin(\omega t)\\ -1 - a \cdot \cos(\omega t) \cdot \sin(\omega t) & -1 + a \cdot \sin^2(\omega t)\end{bmatrix} \cdot x(t) + \begin{bmatrix}0\\1\end{bmatrix} \cdot u(t) \tag{23}$$

$$y(t) = [1 \quad 0] \cdot x(t)$$

where $\omega = 2\pi$ and $\alpha = 1.2$.

Note that we choose $\alpha = 1.2$ to compare the result to that of Sinha and Joseph.[1]

The fundamental matrix $\Phi(t, 0)$ of this system can be written as

$$\Phi(t,0) = \begin{bmatrix} e^{(\alpha-1)\omega t}\cos(\omega t) & e^{-\omega t}\sin(\omega t)\\ -e^{(\alpha-1)\omega t}\sin(\omega t) & e^{-\omega t}\cos(\omega t)\end{bmatrix} = P(t) \cdot e^{\bar{A}t}$$

where the Lyapunov–Floquet transformation matrix is

$$P(t) = \begin{bmatrix}\cos(\omega t) & \sin(\omega t)\\ -\sin(\omega t) & \cos(\omega t)\end{bmatrix}, \qquad e^{\bar{A}t} = \begin{bmatrix} e^{(\alpha-1)\omega t} & 0\\ 0 & e^{-\omega t}\end{bmatrix}$$

Based on the Lyapunov–Floquet transformation, we can get the time-invariant system of the original time-varying system:

$$\begin{aligned}\dot{z}(t) &= \begin{bmatrix}\omega(\alpha-1) & 0\\ 0 & -\omega\end{bmatrix} \cdot z(t) + \begin{bmatrix}-\sin(\omega t)\\ \cos(\omega t)\end{bmatrix} \cdot u(t)\\ yt &= [\cos(\omega t) \quad \sin(\omega t)] \cdot z(t)\end{aligned} \tag{24}$$

And let us define the ideal system as

$$\begin{aligned}\dot{z}^*(t) &= \begin{bmatrix}\omega(\alpha-1) & 0\\ 0 & -\omega\end{bmatrix} \cdot z^*(t) + \begin{bmatrix}1\\1\end{bmatrix} \cdot u^*(t)\\ y^*(t) &= [1 \quad 1] \cdot z^*(t)\end{aligned} \tag{25}$$

which is controllable and observable.

Suppose that we use the pole placement approach to design the controller and estimator of the system, which means we shall decide the constant gain matrices $\bar{G}$ and $\bar{K}$ so that the closed-loop poles of $\bar{A} - \bar{B}\bar{G}$ and $\bar{A} - \bar{K}\bar{C}$ are located at the left half plane of the $j\omega$ axis. If we put $\bar{G} = [g_1 \ \ 0]$ and $\bar{K} = [k_1 \ \ 0]$, then the stability condition of g_1 and k_1 will be $g_1 > \omega(\alpha - 1) = 1.256$ and $k_1 > \omega(\alpha - 1) = 1.256$, respectively. (Note that this stability condition is just for the ideal system, not for the original time-varying system.) To find the stability condition of the original system, we need to check the matrices $\bar{A} - \bar{B}(t)\bar{G}(t)$ and $\bar{A} - \bar{K}(t)\bar{C}(t)$.

By substituting $\bar{G}(t) = \bar{B}^{\#}(t)\bar{B}\bar{G} = \{g_1[\cos(\omega t) - \sin(\omega t)] \ \ 0\}$, we obtain

$$\bar{A} - \bar{B}(t)\bar{G}(t) = \begin{Bmatrix}\omega(\alpha-1) + g_1\sin(\omega t) \cdot [\cos(\omega t) - \sin(\omega t)] & 0\\ g_1\cos(\omega t) \cdot [\cos(\omega t) - \sin(\omega t)] & -\omega\end{Bmatrix}$$

where $g_1 \cos(\omega t) \cdot [\cos(\omega t) - \sin(\omega t)]$ is bounded.

Therefore to make the matrix $\bar{A} - \bar{B}(t)\bar{G}(t)$ stable, the first term of the matrix $\omega(\alpha - 1) + g_1 \sin(\omega t) \cdot [\cos(\omega t) - \sin(\omega t)]$ must be bounded, which is $g_1 > 2\omega(\alpha - 1) = 2.412$. Similarly, for k_1, $k_1 > 2\omega(\alpha - 1) = 2.412$.

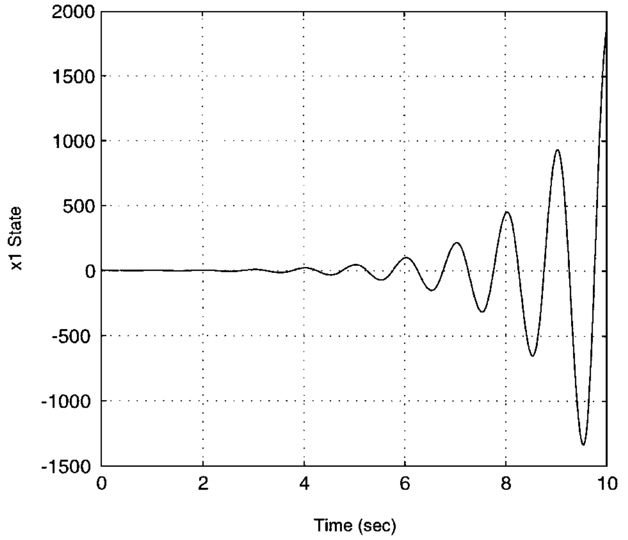

Fig. 1 Step response of the time-varying commutative system ($\bar{G} = \bar{K} = [1.2699\ \ 0]$).

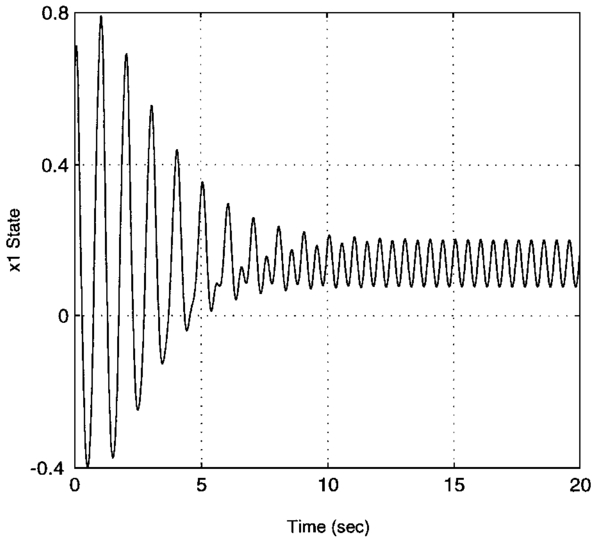

Fig. 2 Step response of the time-varying commutative system ($\bar{G} = \bar{K} = [3.7699\ \ 0]$).

Figures 1 and 2 show that the time response of the system depends on the value of g_1 and k_1, respectively.

V. Conclusion

This Note is devoted to the design of the time-varying controller and estimator of the linear periodic time-varying system. New stability conditions have been developed based on Singh and Joseph's algorithm. The effectiveness of the result was demonstrated by numerical example where the controller and estimator design for the commutative time-varying system has been successfully achieved and the stability of the whole closed-loop system with different values of gain matrices considered.

Acknowledgment

The authors would like to thank the National Renewable Energy Laboratory for its support.

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Adaptive Control of Free-Flying Space Robot with Position/Attitude Control System

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Introduction

SEVERAL investigators have studied the free-flying space robot consisting of a satellite vehicle and manipulators.[1–4] All of the studies have discussed only the case when the manipulators do not handle any objects, or the payloads are known without uncertainty. Robust stability of those controllers has not been studied against the kinematical and dynamical variation. The performance becomes worse when the manipulators handle payloads with uncertain inertia parameters.[5] Murotsu et al. have proposed two parameter identification methods,[5] where a two-stage procedure is necessary, i.e., a controller must use the estimated parameters after the identification. Slotine and Li[6] proposed an adaptive control for the unknown payload manipulation on the ground, but it cannot be applied directly to space robots. Yamamoto et al.[7] and Iwata et al.[8] proposed adaptive controllers for space robots. The former is applicable under the translational and angular momentum conservation whereas the latter does not discuss the stability. Based on Slotine's method, this Note proposes a dynamics-based adaptive controller for the space robot with a position/attitude control system when the robot manipulates a payload.

Equations of Motion

In general, the equations of motion of a space robot can be derived as

$$\boldsymbol{M}\dot{\boldsymbol{u}} + \boldsymbol{h} = \boldsymbol{n} \tag{1}$$

where $\boldsymbol{u} = (\boldsymbol{v}_0^T \boldsymbol{\omega}_0^T \dot{\boldsymbol{\theta}}^T)^T$ and $\boldsymbol{n} = (\boldsymbol{f}_0^T \boldsymbol{\tau}_0^T \boldsymbol{\tau}_\theta^T)^T$. $\boldsymbol{M}$ is an inertia matrix, and $\boldsymbol{h}$ denotes the centrifugal and Coriolis forces. The variables $\boldsymbol{v}_0$ and $\boldsymbol{\omega}_0$ are the translational and the angular velocities of the satellite, respectively, and $\boldsymbol{\theta}$ is the vector of joint positions of the manipulators. The generalized forces $\boldsymbol{f}_0$ and $\boldsymbol{\tau}_0$ represent the translational and the rotational forces applied to the satellite, respectively. $\boldsymbol{\tau}_\theta$ denotes the joint forces. Formally the same equation as Eq. (1) is obtained by the Newton–Euler method and the Lagrangian method, though this study formulates the equations for numerical simulations by Kane's method in the same manner as Yamada et al.[9] If the satellite attitude variable $\boldsymbol{\phi}_0$ satisfies $\boldsymbol{\omega}_0 = \dot{\boldsymbol{\phi}}_0$, Eq. (1) can be rearranged as follows where $\dot{\boldsymbol{q}} = \boldsymbol{u}$:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{h} = \boldsymbol{n} \tag{2}$$

The unknown parameters $\boldsymbol{\beta}$ to be estimated are the inertia parameters of the combined body of the hand and the payload manipulated by the robot. The left side of Eq. (2) can be linearized with respect to $\boldsymbol{\beta}$ as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} = \boldsymbol{P}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\beta} + \boldsymbol{Q} \tag{3}$$

where $\boldsymbol{P}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ called the regressor is a matrix function of $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, and $\ddot{\boldsymbol{q}}$. The first $\dot{\boldsymbol{q}}$ in $\boldsymbol{P}$ is the vector of independent variables in $\boldsymbol{C}$, and

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Received Aug. 8, 1997; presented as Paper 97-3557 at the AIAA Guidance, Navigation, and Control Conference, New Orleans, LA, Aug. 11–13, 1997; revision received Sept. 10, 1998; accepted for publication Jan. 7, 1999.

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